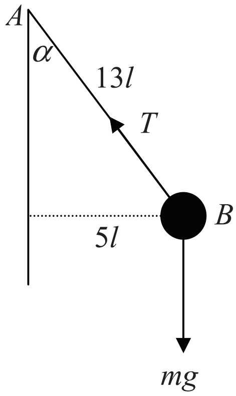
WME03/01: Mechanics M3

Question Number	Scheme	Marks
Q1		
(a)	$\cos\alpha = \dfrac{12}{13}$	B1
	$\text{R}(\uparrow)\quad T\cos\alpha = mg$	M1
	$T \times \dfrac{12}{13} = mg$	
	$T = \dfrac{13}{12}mg$ oe	A1 (3)
(b)	Eqn of motion $\quad T\sin\alpha = m\dfrac{v^2}{5l}$	M1 A1
	$\dfrac{13mg}{12} \times \dfrac{5}{13} = m\dfrac{v^2}{5l}$	M1 dep
	$v^2 = \dfrac{25gl}{12}$	
	$v = \dfrac{5}{2}\sqrt{\dfrac{gl}{3}} \quad \left(\text{accept } 5\sqrt{\dfrac{gl}{12}} \text{ or } \sqrt{\dfrac{25gl}{12}} \text{ or any other equiv}\right)$	A1 (4)
		[7]

Question Number	Scheme	Marks
Q2 (a)	$F = (-)\dfrac{k}{x^2}$	M1
	$mg = (-)\dfrac{k}{R^2}$	M1
	$F = \dfrac{mgR^2}{x^2}$ *	A1 (3)
(b)	$m\ddot{x} = -\dfrac{mgR^2}{x^2}$	M1
	$v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\dfrac{gR^2}{x^2}$	M1
	$\dfrac{1}{2}v^2 = \int\left(-\dfrac{gR^2}{x^2}\right)\mathrm{d}x$	M1 dep on 1st M mark
	$\dfrac{1}{2}v^2 = \dfrac{gR^2}{x} \quad (+c)$	A1
	$x = R,\ \ v = 3U \qquad \dfrac{9U^2}{2} = gR + c$	M1 dep on 3rd M mark
	$\dfrac{1}{2}v^2 = \dfrac{gR^2}{x} + \dfrac{9U^2}{2} - gR$	
	$x = 2R,\ v = U \qquad \dfrac{1}{2}U^2 = \dfrac{gR^2}{2R} + \dfrac{9U^2}{2} - gR$	M1 dep on 3rd M mark
	$U^2 = \dfrac{gR}{8}$	
	$U = \sqrt{\dfrac{gR}{8}}$	A1 (7)
		[10]

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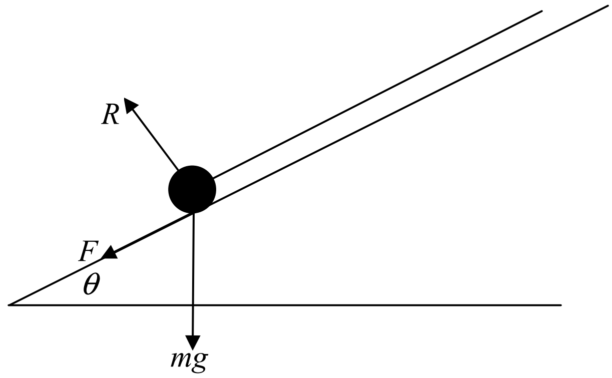
Q3		
	EPE lost $=\dfrac{\lambda\times0.6^2}{2\times0.9}-\dfrac{\lambda\times0.1^2}{2\times0.9}\left(=\dfrac{7}{36}\lambda\right)$	M1 A1
	$\mathrm{R}(\uparrow)\quad R=mg\cos\theta$	M1
	$=0.5g\times\dfrac{4}{5}=0.4g$	
	$F=\mu R=0.15\times0.4g$	M1 A1
	P.E. gained = E.P.E. lost − work done against friction	
	$0.5\mathrm{g}\times0.7\sin\theta=\dfrac{\lambda\times0.6^2}{2\times0.9}-\dfrac{\lambda\times0.1^2}{2\times0.9}-0.15\times0.4g\times0.7$	M1 A1 A1
	$0.1944\lambda=0.5\times9.8\times0.7\times\dfrac{3}{5}+0.15\times0.4\times9.8\times0.7$	
	$\lambda=12.70......$	
	$\lambda=13\text{ N}\quad$ or 12.7	A1
		[9]

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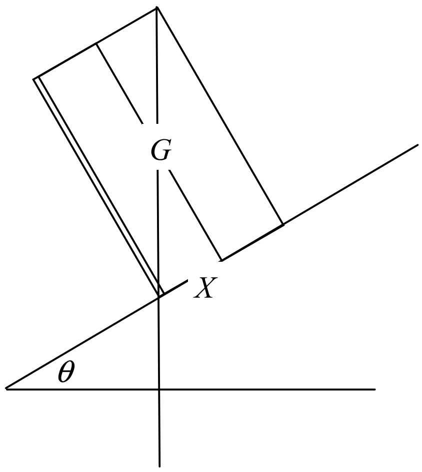
<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>Q4 (a)</td><td>
<table>
<tr><td></td><td>cone</td><td>container</td><td>cylinder</td></tr>
<tr><td>mass ratio</td><td>$\frac{4\pi l^3}{3}$</td><td>$\frac{68\pi l^3}{3}$</td><td>$24\pi l^3$</td></tr>
<tr><td></td><td>4</td><td>68</td><td>72</td></tr>
<tr><td>dist from O</td><td>l</td><td>$\bar{x}$</td><td>$3l$</td></tr>
</table>

Moments: $4l + 68\bar{x} = 72 \times 3l$
$\bar{x} = \frac{212l}{68} = \frac{53}{17}l$ accept $3.12l$
</td><td>M1 A1

B1

M1 A1ft
A1 (6)</td></tr>
<tr><td>(b)</td><td>


$GX = 6l - \bar{x}$ seen
$\tan\theta = \frac{2l}{6l - \bar{x}}$
$= \frac{2 \times 17}{49}$
$\theta = 34.75... = 34.8$ or 35
</td><td>M1
M1 A1

A1 (4)
[10]</td></tr>
</table>

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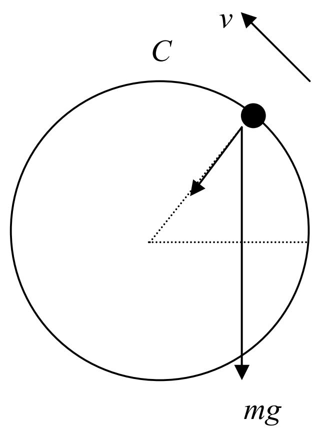
Question Number	Scheme	Marks
Q5	 	
(a)	Energy: $mga\sin\theta = \frac{1}{2}m \times 5ag - \frac{1}{2}mv^2$ $v^2 = 5ag - 2ag\sin\theta$	M1 A1 A1 (3)
(b)	Eqn of motion along radius: $T + mg\sin\theta = \frac{mv^2}{a}$ $T = \frac{m}{a}(5ag - 2ag\sin\theta) - mg\sin\theta$ $T = mg(5 - 3\sin\theta)$	M1 A1 M1 A1 (4)
(c)	At C, $\theta = 90°$ $T = mg(5-3) = 2mg$ $T > 0 \quad \therefore P$ reaches C	M1 A1 A1 (3)
(d)	Max speed at lowest point ($\theta = 270°$; $v^2 = 5ag - 2ag\sin 270$) $v^2 = 5ag + 2ag$ $v = \sqrt{(7ag)}$	M1 A1 (2) **[12]**

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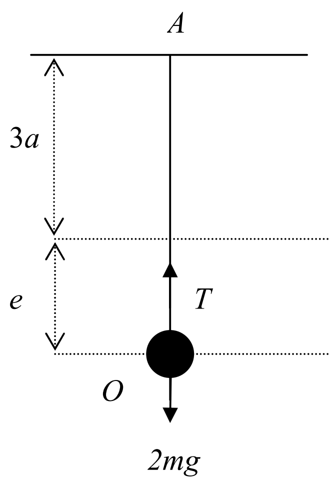
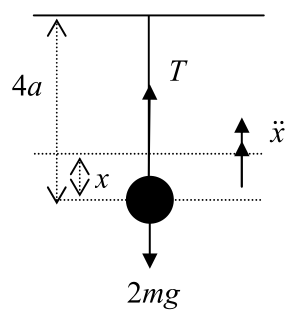
Question Number	Scheme	Marks
Q6 **(a)**	$\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\dfrac{3}{(t+1)^2}$	M1
	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = \int -3(t+1)^{-2}\,\mathrm{d}t$	
	$= 3(t+1)^{-1}\,(+c)$	M1 A1
	$t=0,\ v=2\quad 2=3+c\quad c=-1$	M1
	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{3}{t+1} - 1$ *	A1 **(5)**
(b)	$x = \int \left(\dfrac{3}{t+1} - 1 \right) \mathrm{d}t$	M1
	$= 3\ln(t+1) - t\quad (+c')$	A1
	$t=0,\ x=0\quad \Rightarrow c'=0$	B1
	$x = 3\ln(t+1) - t$	
	$v = 0 \Rightarrow \dfrac{3}{t+1} = 1$	M1
	$t = 2$	A1
	$x = 3\ln 3 - 2$	M1
	$= 1.295...$	
	$= 1.30 \text{ m}\quad$ (Allow 1.3)	A1 **(7)** **[12]**

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>Q7

(a)</td><td>
$\mathrm{R}(\uparrow)\quad T = 2mg$
Hooke's law: $T = \dfrac{6mge}{3a}$
$2mg = \dfrac{6mge}{3a}$
$e = a$
$AO = 4a$</td><td>B1
M1
A1 (3)</td></tr>
<tr><td>(b)</td><td>
H.L. $T = \dfrac{6mg(a-x)}{3a} = \dfrac{2mg(a-x)}{a}$
Eqn. of motion $-2mg + T = 2m\ddot{x}$
$-2mg + \dfrac{2mg(a-x)}{a} = 2m\ddot{x}$
$-\dfrac{2mgx}{a} = 2m\ddot{x}$
$\ddot{x} = -\dfrac{g}{a}x$
period $2\pi\sqrt{\dfrac{a}{g}}$ *</td><td>B1ft
M1
M1
A1
A1 (5)</td></tr>
</table>

Question Number	Scheme	Marks
(c)	$v^2 = \omega^2\left(a^2 - x^2\right)$	
	$v_{\max}{}^2 = \dfrac{g}{a}\left(\left(\dfrac{a}{4}\right)^2 - 0\right)$	M1 A1
	$v_{\max} = \dfrac{1}{4}\sqrt{(ga)}$	A1 (3)
(d)	$x = -\dfrac{a}{8}$ $v^2 = \dfrac{g}{a}\left(\dfrac{a^2}{16} - \dfrac{a^2}{64}\right)$	M1
	$= \dfrac{3ag}{64}$	
	$v^2 = u^2 + 2as$	M1
	$0 = \dfrac{3ag}{64} - 2gh$	A1
	$h = \dfrac{3a}{128}$	
	Total height above $O = \dfrac{a}{8} + \dfrac{3a}{128} = \dfrac{19a}{128}$	A1 (4)
		[15]